**MATHS SPECIALIST UNIT 4**

**INVESTIGATION 4**

**SIMPLE HARMONIC MOTION**

**No notes or calculators allowed in this validation.**

**Time Allowed: 55 minutes Total Marks: 45**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

You should be familiar with the relationship between displacement, velocity and acceleration:

Displacement Velocity Acceleration

  

A special type of motion, called simple harmonic motion, occurs when an object oscillates on a straight line and at any time its acceleration is related to its position x by the differential equation:

, for some constant n and , where x is the distance of the body from the origin (x=0) and at points  the body is stationary (that is v=0).



**Part 1: Acceleration**

**1. [1, 2, 2 = 5 marks]**

Using the fact that , determine the magnitude and direction of the acceleration for the object at:

i) x = 0

ii) x = A

iii) x = -A

**2. [1, 1, 1 = 3 marks]**

Use the information from question 1 to complete the following:

i) The acceleration always points towards the ………………………………………………………..

ii) The magnitude of the acceleration is at a maximum at the ………………………………………

iii) The magnitude of the acceleration is at a minimum at the ………………………………………..

**Part 2: Velocity**

Using the differential equation , we can solve this to obtain an expression for the velocity of an object in simple harmonic motion.



**3. [6 marks]**

Complete the solution to the differential equation above to show that . (Remember that at points  the body is stationary, that is v=0)



**Part 3: Displacement**

Using the differential equation , we can solve this to obtain an expression for the

displacement of an object in simple harmonic motion.

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**4. [9 marks]**

Use the substitution  to solve the differential equation above to show the solution to the

integral is , where = constant of integration.

**5. [2 marks]**

The displacement , can be differentiated to give another expression for the velocity. Determine 

In conclusion for a body in simple harmonic motion we know:

i) ,

for some constant n and , where x is the distance of the body from the origin (x=0) and at points  the body is stationary (that is v=0).

ii) 



iii) 

Use these to answer the following questions:

**6. [3, 2, 2 = 7 marks]**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: , where x is the distance of the body from the origin x=0 and also the body is stationary at points .

1. Hence give an equation for the velocity of the body in terms of *x*, its position on the number line.
2. Given that: when t=0 the body is at x= +2.5 and the velocity is positive give the equation in the form  which shows the position x as a function time.
3. Alternatively: when t=0 the body is at x= +2.5 and the velocity is negative give the equation in the form  which shows the position x as a function time.

**7. [3, 8, 2 = 13 marks]**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: 

1. Hence give an equation for the velocity of the body in terms of *x*, its position on the number line.

(b) Use the substitution to show the solution to the integral

 is: 

(c) Given that: when t=0 the body is at x= +1.5 and the velocity is positive find the solution to the equation which shows the position x as a function time.